Controlling spatiotemporal chaos in coupled map lattices

KaiEn Zhu[1] and Tianlun Chen[2,1]
[1]Institute of Physics, Nankai University, Tianjin 300071, People's Republic of China
[2]CCAST (World Laboratory), P. O. Box 8730, Beijing, 100080, People's Republic of China
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A simple method is presented for controlling spatiotemporal chaos in coupled map lattices to a homogeneous state. This method can be applied to many kinds of models such as coupled map lattices (CML), one-way open CML (the open-flow model), and globally coupled map. We offer the stability analysis of the homogeneous state. Simple and sufficient conditions are obtained for controlling the above mentioned models. Our theoretical results agree well with numerical simulations.

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I. INTRODUCTION

Coupled map lattices (CML) are often used as a convenient model to study characteristics of real spatiotemporal systems [1,2]. Different models have been proposed according to different kinds of couplings. There are many choices for the coupling. The first choice is to choose symmetric coupling or asymmetric coupling [2]. Much work has been done on the symmetric coupling [3]. An extreme asymmetric case that has attracted much interest is one way-open CML [4] corresponding to open-flow systems. Another choice of coupling is to choose local coupling or global coupling. The models mentioned above are all locally coupled models. The globally coupled map (GCM) was introduced by Kaneko [5,6] for the investigation of certain interesting dynamical properties such as clustering of synchronization.

Suppression of spatiotemporal chaotic behavior is an important subject due to its possible applications in plasma, laser devices, turbulence, chemical and biological systems. Various techniques were proposed. Astakhov *et al.* [7] proposed a method for controlling chain of logistic maps on the basis of the Ott-Grebogi-Yorke (OGY) approach [17]. Auerbach showed how an unsymmetrical CML can be controlled to behave periodically by the distributed controllers at several spatial locations [8]. The feedback pinning technique for controlling the CML was discussed [9,10]. Parmananda *et al.* [11] discussed several techniques based on delayed-feedback methods. Konishi *et al.* [12] described a decentralized delayed-feedback control for a one-way open CML.

In the present paper we use a method that was used earlier by de Sousa Vieira and Lichtenberg [13] and later by Huang [14] to control coupled map lattices. Our results show that this method works successfully in spatiotemporal chaotic systems as well as in low-dimensional temporal chaotic systems. We successfully controlled spatiotemporal chaos in several kinds of models: symmetric coupled CML, asymmetric coupled CML (including one-way open CML), and GCM. Stability analysis was presented for the homogeneous state where all sites are equal to the fixed point of the local map. We obtained simple and sufficient conditions for controlling the systems. Numerical simulations agree well with theoretical results. Our method has the following advantages: (1) spatiotemporal chaos can be controlled without any prior knowledge of the local map, (2) the controlling for every site does not need information from other sites.

II. CONTROLLING SPATIOTEMPORAL CHAOS TO THE HOMOGENEOUS STATE

In the present paper we discuss both CML and GCM following by Kaneko [2,5]

$$x_{n+1}^{i}=(1-\varepsilon)f(x_{n}^{i})+\varepsilon[(1-\alpha)f(x_{n}^{i-1})+\alpha f(x_{n}^{i+1})], \tag{1}$$

$$x_{n+1}^{i}=(1-\varepsilon)f(x_{n}^{i})+\frac{\varepsilon}{N}\sum_{j=1}^{N}f(x_{n}^{j}), \tag{2}$$

where n is the discrete time step, i and j are the lattice sites, x_n^i is the system state, N is the system size, $\varepsilon\in(0,1)$ is the coupling strength, $\alpha\in[0,1]$ is a parameter controlling the symmetry of coupling, and $f(x)=1-ax^2$ is the logistic map. Parameter a is fixed to 1.9 in numerical simulations of this paper.

Equation (1) is a quite general model. Let $\alpha=\frac{1}{2}$ we get the ordinary symmetric coupled CML, otherwise we get asymmetric coupled CML. An extreme case of asymmetric coupling is one-way open CML by choosing $\alpha=0$.

Our goal is to control spatiotemporal chaos in CML and GCM to a homogeneous state

$$(x_n^1,x_n^2,\ldots,x_n^N)^T=(x_f,x_f,\ldots,x_f)^T,$$

where x_f is the fixed point of the local map f, that is, $x_f=f(x_f)$. In order to do this, we add a control term that is based on the controlling method of de Sousa Vieira and Lichtenberg [13] and Huang [14] to the right-hand side of Eq. (1), Eq. (2),

$$x_{n+1}^{i}=(1-\varepsilon)f(x_{n}^{i})+\varepsilon[(1-\alpha)f(x_{n}^{i-1})+\alpha f(x_{n}^{i+1})]-\gamma(f(x_n^i)-x_n^i), \tag{3}$$

$$x_{n+1}^{i}=(1-\varepsilon)f(x_{n}^{i})+\frac{\varepsilon}{N}\sum_{j=1}^{N}f(x_{n}^{j})-\gamma(f(x_n^i)-x_n^i) \tag{4}$$

It is obvious that the control term does not change the fixed points of the original systems.

Next we will analyze the stability of Eq. (3) and Eq. (4) to show that the homogeneous state becomes stable when the controlling term is switched on.

A. The stability analysis of CML

The stability of Eq. (3) at the homogeneous state depends on the eigenvalues of its Jacobian matrix. Since the boundary conditions affect the Jacobian matrix, we choose a fixed boundary conditions in this section in order to simplify theoretical analysis. Fixed boundary condition means that x_n^0 and x_n^{N+1} are fixed at $x_n^0 = x_n^{N+1} = x_f$. The effect of boundary conditions will be discussed in the Sec. III.

From Eq. (3) we have

$$\frac{\partial x_{n+1}^i}{\partial x_n^j} = \begin{cases} \varepsilon(1-\alpha)\Lambda & \text{for } j=i-1 \\ (1-\varepsilon)\Lambda - \gamma(\Lambda-1) & \text{for } j=i \\ \varepsilon\alpha\Lambda & \text{for } j=i+1 \\ 0 & \text{for } |j-i|>1, \end{cases}$$

where $\Lambda \equiv [\partial f(x)]/\partial x|_{x=x_f}$. Let $A \equiv (1-\varepsilon)\Lambda - \gamma(\Lambda-1)$, $B \equiv \varepsilon(1-\alpha)\Lambda$, $C \equiv \varepsilon\alpha\Lambda$ and denote $M(X_f)$ as the Jacobian matrix of Eq. (3) at the homogeneous state, we have

$$M(X_f) = \begin{pmatrix} A & C & 0 & \cdots & 0 & 0 \\ B & A & C & \cdots & 0 & 0 \\ 0 & B & A & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & A & C \\ 0 & 0 & 0 & \cdots & B & A \end{pmatrix}.$$

The deduction of the eigenvalues of $M(X_f)$ can be found in books for numerical solution of partial differential equations [15]. The eigenvalues of Jacobian matrix $M(X_f)$ is,

$$\lambda_i = A + 2\sqrt{BC}\cos\frac{i\pi}{N+1}, \quad \text{where } i=1,2,\ldots,N.$$

When N is large,

$$\lambda_{min} \equiv \min(\lambda_1,\lambda_2,\ldots,\lambda_N) \approx A - 2\sqrt{BC}, \tag{5}$$

$$\lambda_{max} \equiv \max(\lambda_1,\lambda_2,\ldots,\lambda_N) \approx A + 2\sqrt{BC}. \tag{6}$$

A stable homogeneous state requires $|\lambda_i|<1$ for all i. This requirement is an equivalence of

$$\begin{cases} \lambda_{min} > -1 \\ \lambda_{max} < 1. \end{cases} \tag{7}$$

Since $\Lambda < (-1)$ for logistic map, Eq. (5), Eq. (6), and Eq. (7) lead to

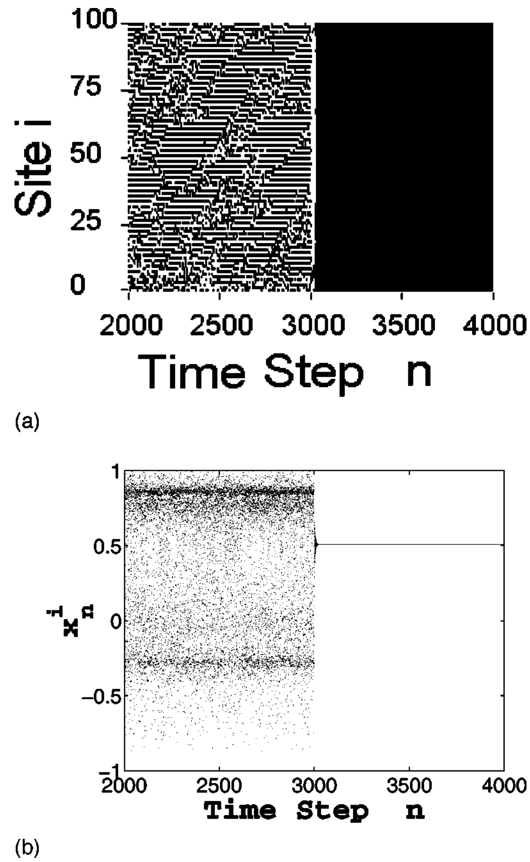

FIG. 1. Controlling of asymmetric coupled CML with $a=1.9$, $\varepsilon=0.1$, $\gamma=0.38$, $\alpha=0.25$, $N=100$ starting from a random initial condition. Controlling is switched on at $n=3000$. (a) Site-time step diagram. Pixels are painted black if $x_n^i > 0.508$, and white otherwise. Every eighth step is plotted along the time axis. (b) Amplitude-time step diagram. The values of all sites are plotted, that is, $i=1,2,\ldots,100$ in x_n^i.

$$\frac{-1-(1-\varepsilon)\Lambda + 2\varepsilon|\Lambda|\sqrt{\alpha(1-\alpha)}}{1-\Lambda} < \gamma < \frac{1-(1-\varepsilon)\Lambda - 2\varepsilon|\Lambda|\sqrt{\alpha(1-\alpha)}}{1-\Lambda}. \tag{8}$$

This is a sufficient condition for a stable homogeneous state.

Once we know the values of a and ε, we can choose controlling parameter γ according to the above inequality (8). For example, if $a=1.9$, $\varepsilon=0.1$, and $\alpha=0.25$, inequality (8) means $0.3092<\gamma<0.8770$. This means that once γ is chosen in $(0.3092, 0.8770)$, the homogeneous state becomes stable when the controlling term is switched on. Figure 1 shows the numerical simulations with the controlling parameter $\gamma=0.38$. After the controlling term is switched on, the spatiotemporal chaotic behavior in the asymmetric coupled CML soon turns to the homogeneous state.

Inequality (8) gives the way to choose appropriate controlling parameter for successfully controlling spatiotemporal chaos. A better understanding of inequality (8) is a graph. The region between the two lines in Fig. 2 is the stable region satisfying inequality (8). The result of numerical

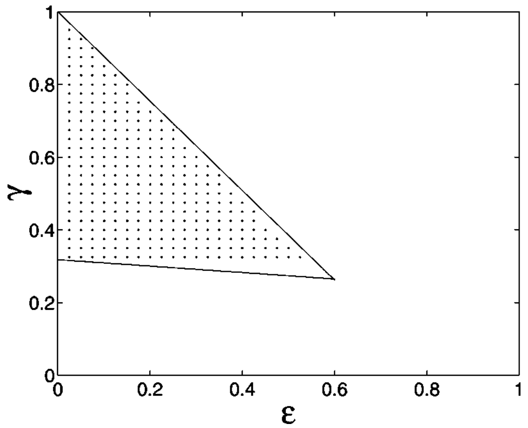

FIG. 2. Stable region of asymmetric coupled CML for the homogeneous state. The region between the two lines is the stable region of the theoretical result. The dots in the figure are the parameters that were controlled to the homogeneous state by numeral simulation. Calculation was carried out by dividing the (0,1) $\times$(0,1) region of the $\gamma-\varepsilon$ plane into 40$\times$40 grid for calculation. The other parameters are $a=1.9$, $\alpha=0.25$, $N=100$. The initial condition is $0.8\sin(2\pi(i-1)/N)$. Controlling is switched on at $n=2500$. Data are obtained after $n=9990$.

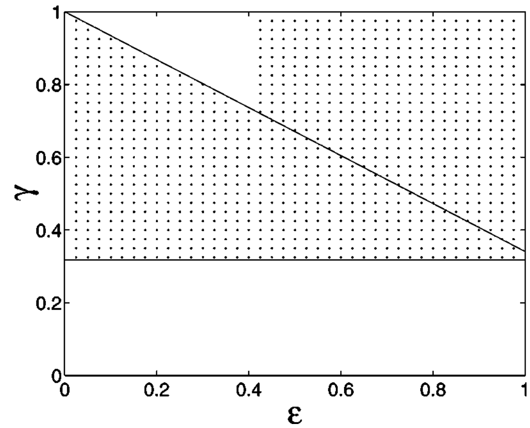

FIG. 3. Stable region of GCM for the homogeneous state. The region between the two lines is the stable region of the theoretical result. The dots in the figure are the parameters that were controlled to the homogeneous state. Parameters are the same as Fig. 2 except α, which is not used in GCM.

simulations is also showed in the same figure. The dots in the figure are the parameters that were controlled to the homogeneous state. Theoretical result agrees well with numerical simulation for the asymmetric coupled CML with $\alpha=0.25$.

We have showed that our controlling method works well for a asymmetric coupled CML with $\alpha=0.25$. Controlling for other cases are also successful. However, in consideration of brevity we would not present the numerical results of other cases such as symmetric coupled CML, one-way open CML, etc.

B. The stability analysis of GCM

The stability of Eq. (4) at the homogeneous state depends on the eigenvalues of its Jacobian matrix. From Eq. (4) we have

$$\frac{\partial x_{n+1}^i}{\partial x_n^j}=\begin{cases}(1-\varepsilon)\Lambda-\gamma(\Lambda-1)+\dfrac{\varepsilon}{N}\Lambda & \text{for } j=i\\[2ex] \dfrac{\varepsilon}{N}\Lambda & \text{for } j\neq i,\end{cases}$$

where $\Lambda\equiv[\partial f(x)]/\partial x|_{x=x_f}$. Let $A\equiv(1-\varepsilon)\Lambda-\gamma(\Lambda-1)$, $B\equiv\varepsilon/N\Lambda$, and denote $M(X_f)$ as the Jacobian matrix of Eq. (4) at the homogeneous state, we have

$$M(X_f)=\begin{pmatrix} A+B & B & B & \cdots & B\\ B & A+B & B & \cdots & B\\ B & B & A+B & \cdots & B\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ B & B & B & \cdots & A+B\end{pmatrix}.$$

The characteristic polynomial of $M(X_f)$ is $|\lambda I-M(X_f)|$. It is easy to calculate it (We can first multiply the last row by (-1) and add it to other lines. Then we add column $1,2,\ldots,N-1$ to column N.) So the eigenvalues of $M(X_f)$ are

$$\lambda_1=\lambda_2=\cdots=\lambda_{N-1}=A=(1-\varepsilon)\Lambda-\gamma(\Lambda-1) \tag{9}$$

and

$$\lambda_N=A+NB=\Lambda-\gamma(\Lambda-1). \tag{10}$$

A stable homogeneous states means that $|\lambda_i|<1$ for all i. So we get

$$\begin{cases}|(1-\varepsilon)\Lambda-\gamma(\Lambda-1)|<1\\ |\Lambda-\gamma(\Lambda-1)|<1.\end{cases} \tag{11}$$

We choose $\varepsilon\in(0,1)$. For logistic map, $\Lambda<-1$. Solving Eq. (11) under these two conditions, we have

$$\frac{-1-\Lambda}{1-\Lambda}<\gamma<\frac{1-(1-\varepsilon)\Lambda}{1-\Lambda}. \tag{12}$$

This is the condition of stable homogeneous state of GCM. The parameter satisfying the above inequality is the area between the two lines in Fig. 3.

Figure 3 also shows the numerical result of GCM. The dots in the figure are the parameters that were controlled to stable homogeneous state. An interesting thing is that we got an extra stable region that locates outside the theoretical stable region. The reason is the following. When ε is large, the system soon reaches a state where all sites are equal before the controlling term is switched on. This state was called coherent phase by Kaneko [5]. The sites in this state act as uncoupled maps. In this simple uncoupled case, our controlling method is the same as those in Refs. [13] and [14]. Every site is controlled to the fixed point with no interaction from other sites.

III. DISCUSSION AND CONCLUSION

Numerical results show that our controlling method can control spatiotemporal chaos in different boundary conditions such as fixed boundary condition, periodic boundary condition, etc. Since Jacobian matrix is slightly different for different boundary conditions, the eigenvalues vary for different boundary conditions. A useful tool to estimate eigenvalues is the *Geršgorin* disks theorem [16] in matrix theory. It tells us that all the eigenvalues of a matrix locate in some regions. We require that all the regions lie in $(-1,1)$. This requirement leads to an inequality just like inequality (8) for successful controlling.

The *Geršgorin* disks theorem also shows that as long as a boundary does not give out large stimulation, it does not change eigenvalues of Eq. (3) and Eq. (4) in $(-1,1)$ to the region outside of $(-1,1)$. Since many boundaries do not give out large stimulation comparing to the coupling, our controlling method works well in these boundary conditions.

We successfully use a simple method to control symmetric coupled CML, asymmetric coupled CML, and globally coupled map (GCM). Simple and sufficient conditions are obtained for controlling these models to the homogeneous state.

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